EXERCISE – II — HINTS & SOLUTIONS

Sol.1 C

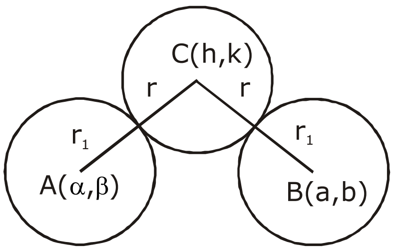
$CA - r_1 = r$

$CB - r_2 = r$

$CA - CB = r_1 - r_2 = k$

$CA - CB = k$

$\Rightarrow$ Locus of C will be hyperbola.



Sol.2 C

Let M (h, k)

Chord with given mid point (h, k)

$$T = S_1 \Rightarrow \frac{hx}{a^2} - \frac{ky}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

$$(\alpha, \beta) \Rightarrow \frac{h\alpha}{a^2} - \frac{k\beta}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$$

$$\frac{x\alpha}{a^2} - \frac{y\alpha}{b^2} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

$$\frac{x^2}{a^2} - \frac{x\alpha}{a^2} - \left(\frac{y^2}{b^2} - \frac{y\beta}{b^2}\right) = 0$$

$$\frac{x^2}{a^2} - \frac{x\alpha}{a^2} + \frac{\alpha^2}{4a^2} - \frac{\alpha^2}{4a^2} - \left(\frac{y^2}{b^2} - \frac{y\beta}{b^2} + \frac{\beta^2}{4b^2} - \frac{\beta^2}{4b^2}\right) = 0$$

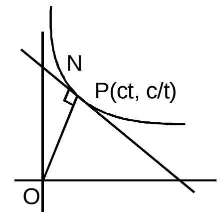
$$\left(\frac{x}{a} - \frac{\alpha}{2a}\right)^2 - \left(\frac{y}{b} - \frac{\beta}{2b}\right)^2 = \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2}$$

Centre will be $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ And Hyperbola

Sol.3 D

$xy = c^2$

$P\left(ct, \frac{c}{t}\right)$



Tangent at P

$\frac{x}{t} + ty = 2c$ ....(1) Let N (h, k)

Slope of ON

$m = t^2 = \frac{k}{h}$ ....(2)

point (h, k) lie on Equation (1)

$\frac{h}{t} + tk = 2c$

$$\Rightarrow (h + t^2k)^2 = 4c^2 t^2 \Rightarrow \left(h + \frac{k^2}{h}\right)^2 = 4c^2 \frac{k}{h}$$

$$\Rightarrow (h^2 + k^2)^2 = 4c^2 kh \Rightarrow (x^2 + y^2) = 4c^2 xy$$

Sol.4 A

Let $P(ct_1, c/t_1)$ $Q(ct_2, c/t_2)$

$$M_{PQ} = \frac{\frac{c}{t_2} - \frac{c}{t_1}}{c(t_2 - t_1)} = \frac{-1}{t_1t_2}$$

$$\text{Equation } y - \frac{c}{t_1} = \frac{-1}{t_1t_2}(x - ct_1)$$

$$x + t_1t_2y = c(t_1 + t_2) \Rightarrow \frac{x}{(ct_1 + ct_2)} + \frac{y}{\left(\frac{c}{t_2} + \frac{c}{t_1}\right)} = 1$$

$$\frac{x}{x_1 + x_2} + \frac{y}{(y_1 + y_2)} = 1$$

Sol.5 C

$9x^2 - 16y^2 - 18x + 32y - 151 = 0$

$9(x^2 - 2x) - 16(y^2 - 2y) - 151 = 0$

$9(x^2 - 2x + 1) - 9 - 16(y^2 - 2y + 1) + 16 - 151 = 0$

$9(x - 1)^2 - 16(y - 1)^2 = 144$

$$\frac{(x-1)^2}{\left(\frac{144}{9}\right)}-\frac{(y-1)^2}{\left(\frac{144}{16}\right)}=1 \Rightarrow \frac{(x-1)^2}{16}-\frac{(y-1)^2}{9}=1$$

$\ell(TA) = 2a = 8$ $\qquad e^2 = 1 + \frac{b^2}{a^2}$

$\Rightarrow e = \frac{5}{4}$

$\ell(LR) = \frac{2b^2}{a} = \frac{2\times 9}{4} = \frac{9}{2}$

Directries $x - 1 = \frac{4}{\left(\frac{5}{4}\right)}$ and $x - 1 = -\frac{16}{5}$

$x = \frac{21}{5}$ $\qquad x = -\frac{11}{5}$

Sol.6 **A**

Let P(a cosθ, a sinθ)

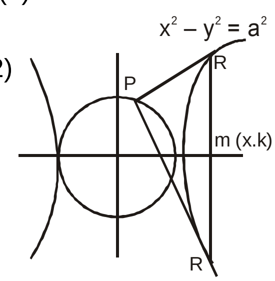
Equation of QR (c.o.c. w.r.t. p) T = 0

$x \cos\theta - y \sin\theta = a$ ...(1)

and $\quad T = S_1$

$hx - ky = h^2 - k^2$ ...(2)

(1) and (2) are same



$$\frac{\cos\theta}{h}=\frac{\sin\theta}{k}=\frac{a}{h^2-k^2}$$

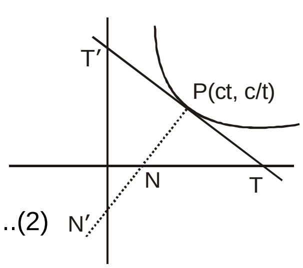
square & add

$(x^2 - y^2)^2 = a^2 (x^2 + y^2)$

Sol.7 **C**

Tangent at P

$\frac{x}{t} + ty = 2c$ ....(1)



Normal at P

$y - \frac{c}{t} = xt^2 - ct^3$ ....(2)

T (2ct, 0) ; T′ (0, 2c/t)

$N\left(ct-\frac{c}{t^3},0\right)$; $N'\left(0,\frac{c}{t}-ct^3\right)$

Δ = Area of ΔPNT = $\frac{1}{2} \times \frac{c}{t}\left[2ct-ct+\frac{c}{t^3}\right]$

$\Delta = \frac{c^2}{2t^4}(t^4 + 1)$

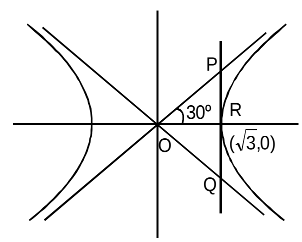
Δ' = Area of ΔPN′T′

$= \frac{1}{2} \times ct \times \left[\frac{2c}{t}-\frac{c}{t}+ct^3\right] = \frac{1}{2} c^2 (t^4 + 1)$

$$\frac{1}{\Delta}+\frac{1}{\Delta'}=\frac{2}{c^2}$$

Sol.8 **B,C**

$\frac{x^2}{3} - \frac{y^2}{1} = 1$



Asyp $y = \pm\frac{1}{\sqrt{3}}x$

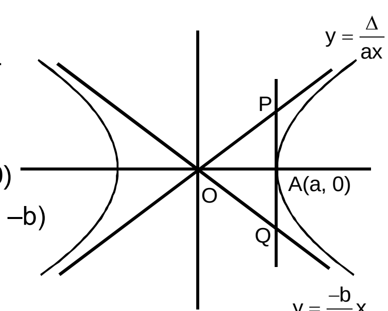
ΔOPQ will be equilateral triangle.

PR = 1

area of ΔOPQ = $\frac{1}{2} \times \sqrt{3} \times (2) = \sqrt{3}$ sq. units

Sol.9 **A**

$\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$



Let the any point be (a, 0)

P(a, b), Q (a, –b)

PQ = 2b

OA = a

Area of ΔOPA = $\frac{1}{2} \times a \times 2b = ab$

$\Rightarrow ab = a^2 \tan\lambda \Rightarrow \frac{b}{a} = \tan\lambda$

$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\tan^2\lambda} = \sec\lambda$

Sol.10 D

Let the point $(a \sec\theta, b\tan\theta)$

C.O.C. : $\dfrac{x}{a}\sec\theta - \dfrac{y}{b}\tan\theta = 2$ ....(1)

PoI of asymptotes and Eq^n (1)

$A[2a(\sec\theta + \tan\theta), 2b(\sec\theta + \tan\theta)$

$B[2a(\sec\theta - \tan\theta), -2b(\sec\theta - \tan\theta)$

Area of Triangle OAB = $\dfrac{1}{2}(8ab) = 4ab$

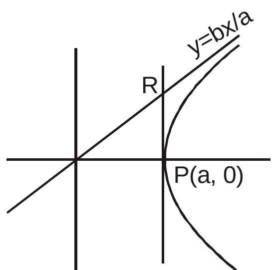
Sol.11 C

P(a, 0) ; Q(a, b)

Let M (h, k)

$2h = 2a \Rightarrow h = a$

$k = \dfrac{b}{2}$



$$\left(\frac{h}{a}\right)^2 - \left(\frac{k}{b}\right)^2 = 1 - \frac{1}{4}$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{3}{4} \quad \text{So } k = \frac{3}{4}$$

Sol.12 A,D

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \text{Asyp. } y = \pm\frac{b}{a}x$$

$$m_1 = \frac{b}{a} \text{ and } m_2 = -\frac{b}{a}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{b}{a} + \frac{b}{a}}{1 - \frac{b^2}{a^2}}\right|$$

$$\tan\theta = \frac{2ab}{a^2 - b^2} \Rightarrow \tan\frac{\theta}{2} = \frac{b}{a} \text{ and } -\frac{a}{b}$$

$$\sec\frac{\theta}{2} = \sqrt{1 + \frac{b^2}{a^2}} \text{ and } \sec\frac{\theta}{2} = \sqrt{1 + \frac{a^2}{b^2}}$$

$$= e \qquad\qquad = \frac{1}{e}$$

Sol.13 A,D

mid pt. of $F_1F_2 \Rightarrow$ Centre of conic

$C\left(\dfrac{29}{2}, \dfrac{19}{2}\right)$

$$\text{equation } \frac{\left(x - \frac{29}{2}\right)^2}{a^2} - \frac{\left(y - \frac{19}{2}\right)^2}{b^2} = 1$$

passing through origin

$$\frac{(29)^2}{4a^2} - \frac{(19)^2}{4b^2} = 1$$

$2ae = \sqrt{386}$

Solve & get e

Sol.14 A,B

Let the point $P(x_1, y_1)$

tangent at P

$xx_1 - 9yy_1 = 9$

$$x\left(\frac{x_1}{9}\right) - y(y_1) = 1 \qquad \text{....(1)}$$

$$\left(\frac{5}{19}\right)x + \left(\frac{12}{19}\right)y = 1 \qquad \text{....(2)}$$

By comparing (1) & (2)

$$x_1 = \frac{45}{19} : y_1 = \frac{-12}{19}$$

Sol.15 B,D

Hyperbola if

$h^2 > ab$

$\Rightarrow \lambda^2 > (2 + \lambda)(\lambda - 1)$

$\Rightarrow \lambda < 2$

and $D \neq 0 \Rightarrow -2[3\lambda - 4] \neq 0$

$\Rightarrow \lambda \neq 4/3$